

Kaluza-Klein brane cosmology with a bulk scalar field

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The brane-world cosmological model in higher-dimensional spacetime with a bulk scalar field is investigated. We derive the $(4+n)$ -dimensional gravitational field equations for the scalar field on the $(3+n)$ -brane in a $(5+n)$ -dimensional bulk with Einstein gravity plus a self-interacting scalar field. The $(4+n)$ -dimensional gravitational field equations can be formulated to standard form with the extra component. Using this formalism we study the Kaluza-Klein brane cosmology. We derive the Friedmann equation and a possible energy leak out of the brane into the bulk. We present some exact solutions corresponding to vacuum brane and matter on the brane.

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I. INTRODUCTION

The idea that our universe has dimensions more than four has been around since the first attempts to unify fundamental forces. The Kaluza-Klein theories were one of the first attempts towards this direction. According to the Kaluza-Klein picture, extra dimensions are compactified to a very small length scale (naturally the Planck scale), and as a result spacetime appears to be effectively four-dimensions, insofar as low energies are concerned. On the other hand, inspired by the discovery of D-branes within string theories, the brane-world scenario is now one of the most important ideas in high energy physics. According to brane-world scenario, our physical Universe is envisioned as a four-dimensional hypersurface in a five-dimensional bulk spacetime. The standard model matter is confined to the brane but gravity, by its universal character, can propagate in the extra dimensions. Much efforts to reveal cosmology on the brane have been done in the context of five-dimensional spacetime, especially after the stimulating proposals by Randall and Sundrum (RS) [1, 2]. In this model, a five-dimensional realization of the Horava-Witten solution [3], the hierarchy problem can be solved by introducing an appropriated exponential warp factor in the metric. The various properties and characteristics of the RS model have been extensively an-

alyzed: the cosmology framework [4–8], the low energy effective theory [9–20], the black hole physics [21–26], and the Lorentz violation [27–37].

The brane-world models with scalar field in the bulk has been discussed by various authors, see [38] and references therein. It is believed that in the unified theory approach, a dilatonic gravitational scalar field term is required in the Einstein-Hilbert action [38]. One of the first motivations to introduce a bulk scalar field is to stabilize the distance between the two branes [39] in the context of the first model introduced by Randall and Sundrum. A second, motivation for studying scalar fields in the bulk is due the possibility that such a setup could provide some clue to solve the famous cosmological constant problem. Models with inflation driven by bulk scalar field have been studied and it is shown that inflation is possible without inflaton on the brane [40]. Later the quantum fluctuations of brane-inflation and reheating issues are also addressed in the braneworld scenario with bulk scalar field [41]. The creation of a brane world with a bulk scalar field using an instanton solution in a five dimensional Euclidean Einstein equations is also considered [42].

In this paper, our main purpose is to construct brane cosmological models in higher-dimensional spacetime with a bulk scalar field. We generalize the case four-dimensional brane models to $(4+n)$ -dimensional brane models where n represents internal dimensions of the brane. In order to obtain four-dimensional description of our Universe, we combine two representative and contrastive approaches, that is, the Randall-Sundrum and the Kaluza-Klein scenario. In the former case, the compactification is done by the localization of the configuration

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along the extra dimension, while in the latter one the reduction is achieved by the compactification of the internal space of the brane. Such a way of construction is called Kaluza-Klein brane-world, and it has been investigated by various authors [43–51].

This paper is organized as follows. In Section II, we study a higher brane-world model in a $(5+n)$ -dimensional spacetime with a bulk scalar field. We derive the $(4+n)$ -dimensional Einstein equations using the geometrical approach. We transform the non-conventional kinetic term in the Einstein equations into the standard form. In Section III, the Kaluza-Klein brane worlds cosmology is presented by using the extra component formalism. Then we discuss the vacuum brane solutions with an initial singularity and a possible energy leak out of the brane into the bulk. For completeness, we study the static internal dimension with matter on the brane in Section IV. We derive the Friedmann equation and a possible energy leak out of the brane into the bulk. In section V, we study dimensional reduction of the higher-dimensional theory and then performing a conformal transformation in order to recover ordinary 4-dimensional general relativity. Section VI is devoted to the conclusions.

II. EFFECTIVE GRAVITATIONAL EQUATIONS

A. Action and field equations

We consider the higher-dimensional dilatonic brane-world, i.e. the higher-dimensional brane-world where the bulk is allowed to contain a single non-gravitational degree of freedom, the scalar field ϕ . A $(3+n)$ -brane with the $(4+n)$ -dimensional spacetime embedded in the $(5+n)$ -dimensional spacetime is located at $y = 0$, where the y direction is compactified on the orbifold. This higher-dimensional dilatonic brane-world model is described by the action

$$S = \int d^{5+n}x \sqrt{-g} \left[\frac{1}{2\kappa^2} \mathcal{R} - \frac{1}{2} g^{ab} \nabla_a \phi \nabla_b \phi - V(\phi) \right] + \int d^{4+n}x \sqrt{-h} [-\sigma(\phi) + \mathcal{L}_m], \quad (1)$$

where \mathcal{R} is the Ricci scalar of the $(5+n)$ -dimensional metric g_{ab} . σ is the brane tension which is allowed to be the function of ϕ , and \mathcal{L}_m is the Lagrangian for matter localized on the brane. A metric $h_{\mu\nu}$ is the induced metric on the brane.

We write the coordinate system for the bulk spacetime in the form

$$ds^2 = g_{ab} dx^a dx^b = dy^2 + g_{\mu\nu}(y, x) dx^\mu dx^\nu, \quad (2)$$

and we may assume that the position of the brane is $y = 0$ in this coordinate system so that the induced metric on the brane is $h_{\mu\nu}(x) = g_{\mu\nu}(y = 0, x)$ and the extrinsic curvature is defined as

$$K_{\mu\nu} = -\frac{1}{2} \frac{\partial}{\partial y} g_{\mu\nu} \equiv -\frac{1}{2} g_{\mu\nu, y}. \quad (3)$$

The Einstein equations can be obtained by varying the action (1) with respect to the gravitational field g_{ab} ,

$$\mathcal{G}_{ab} = \kappa^2 \mathcal{T}_{ab} + \kappa^2 \delta_a^\mu \delta_b^\nu (-\sigma g_{\mu\nu} + t_{\mu\nu}) \delta(y), \quad (4)$$

where $\mathcal{G}_{ab} = \mathcal{R}_{ab} - g_{ab} \mathcal{R}/2$ is the $(5+n)$ -dimensional Einstein tensor, and \mathcal{T}_{ab} is the energy-momentum tensor for the scalar field,

$$\mathcal{T}_{ab} = \nabla_a \phi \nabla_b \phi - g_{ab} \left(\frac{1}{2} g^{cd} \nabla_c \phi \nabla_d \phi + V(\phi) \right), \quad (5)$$

while the total energy-momentum tensor on the brane $t_{\mu\nu}$ is defined as

$$t_{\mu\nu} \equiv -2 \frac{\delta \mathcal{L}_m}{\delta g^{\mu\nu}} + g_{\mu\nu} \mathcal{L}_m. \quad (6)$$

The equation of motion for the scalar field reads:

$$\nabla^a \nabla_a \phi - V'(\phi) - \sigma'(\phi) \delta(y) = 0, \quad (7)$$

where a prime denotes a derivative with respect to ϕ .

In the coordinate system (2) and by the application of the Gauss-Codazzi equations, one has the $(5+n)$ -dimensional field equations as follows

$$\mathcal{G}_y^y = -\frac{1}{2} R + \frac{1}{2} K^2 - \frac{1}{2} K^{\alpha\beta} K_{\alpha\beta} = \kappa^2 \mathcal{T}_y^y, \quad (8)$$

$$\mathcal{G}_\mu^y = -\nabla_\alpha K_\mu^\alpha + \nabla_\mu K = \kappa^2 \mathcal{T}_\mu^y, \quad (9)$$

$$\begin{aligned} \mathcal{G}_\nu^\mu &= G_\nu^\mu + (K_\nu^\mu - \delta_\nu^\mu K)_{,y} - K K_\nu^\mu \\ &\quad + \frac{1}{2} \delta_\nu^\mu (K^2 + K^{\alpha\beta} K_{\alpha\beta}) \\ &= \kappa^2 \mathcal{T}_\nu^\mu + \kappa^2 (-\sigma \delta_\nu^\mu + t_\nu^\mu) \delta(y), \end{aligned} \quad (10)$$

where $G_\nu^\mu = R_\nu^\mu - \delta_\nu^\mu R/2$ is the $(4+n)$ -dimensional Einstein tensor and the covariant derivative ∇_μ is calculated with respect to the metric $g_{\mu\nu}$. Combining equations (8) with (10), and using the $(5+n)$ -dimensional Weyl tensor, we obtain the following $(4+n)$ -dimensional Einstein equations

$$\begin{aligned} G_\nu^\mu &= -K^{\mu\alpha} K_{\alpha\nu} + K K_\nu^\mu + \frac{1}{2} \delta_\nu^\mu (K_{\alpha\beta} K^{\alpha\beta} - K^2) \\ &\quad + \frac{2+n}{3+n} \kappa^2 \left[\mathcal{T}_\nu^\mu - \frac{1}{4+n} \delta_\nu^\mu \mathcal{T}_\alpha^\alpha + \frac{3+n}{4+n} \delta_\nu^\mu \mathcal{T}_y^y \right] \\ &\quad - E_\nu^\mu, \end{aligned} \quad (11)$$

Here, we have defined that the term $\mathcal{T}_\alpha^\alpha$ is the trace defined with respect to the $(4+n)$ -dimensional metric $g_{\mu\nu}$, and the projected Weyl tensor is defined as $E_{\mu\nu} = C_{y\mu y\nu}|_{y=0}$. To eliminate the extrinsic curvature, we need the junction conditions. It can be obtained by collecting together the terms in field equations which contain a δ -function. By assuming Z_2 -symmetry we find

$$[K_\nu^\mu - \delta_\nu^\mu K]|_{y=0} = \frac{\kappa^2}{2} (-\sigma \delta_\nu^\mu + t_\nu^\mu), \quad (12)$$

$$[\partial_y \phi]|_{y=0} = \frac{1}{2} \sigma'(\phi). \quad (13)$$

Using this junction conditions, we find the $(4+n)$ -dimensional generalization of the Shiromizu-Maeda-Sasaki equations [5],

$$G^\mu_\nu = \frac{(2+n)\kappa^4}{4(3+n)}\sigma t^\mu_\nu + \frac{(2+n)\kappa^2}{(3+n)}\left[\nabla^\mu\phi\nabla_\nu\phi - \frac{(5+n)}{2(4+n)}\delta^\mu_\nu\nabla^\alpha\phi\nabla_\alpha\phi\right] - \Lambda_b\delta^\mu_\nu + \kappa^4\pi^\mu_\nu - E^\mu_\nu, \quad (14)$$

where the induced cosmological constant on the brane is given by

$$\Lambda_b(\phi) = \frac{(2+n)\kappa^2}{(4+n)}\left[V + \frac{(4+n)\kappa^2}{8(3+n)}\sigma^2 - \frac{1}{8}\sigma'^2\right], \quad (15)$$

and the local quadratic energy-momentum tensor on the brane is

$$\pi^\mu_\nu = -\frac{1}{4}t^\mu_\alpha t^\alpha_\nu + \frac{1}{4(3+n)}tt^\mu_\nu + \frac{1}{8}\delta^\mu_\nu\left(t_{\alpha\beta}t^{\alpha\beta} - \frac{1}{3+n}t^2\right). \quad (16)$$

We note that Eq. (15) may not be constant in general, as is clear from its expression. The first term comes from the scalar field potential. The second term is contribution from the brane tension, which yields extrinsic curvature of the brane and its quadratic. The third term is the first derivative of the brane tension, which leads to a discontinuity in the scalar field gradient normal to the brane.

The scalar field equation of motion is given by

$$\nabla^\alpha\nabla_\alpha\phi - \frac{1}{\kappa^2}\Lambda'_b(\phi) = J_n, \quad (17)$$

where a possible energy leak out of the brane into the bulk, J_n , is defined by

$$J_n = \frac{(2+n)}{4(4+n)}\left(\sigma'\sigma'' - \frac{\kappa^2}{3+n}\sigma t\right) + \frac{2}{4+n}\nabla^\alpha\nabla_\alpha\phi - \frac{2+n}{4+n}[\partial_y^2\phi]_{y=0}. \quad (18)$$

B. Extra component formalism

The induced Einstein equations on the brane, Eq. (14), contain the non-conventional kinetic term. However, we can transform the non-conventional kinetic term into the standard form, then Eq. (14) can be rewritten as

$$G^\mu_\nu = \kappa^2\left(\nabla^\mu\phi\nabla_\nu\phi - \frac{1}{2}\delta^\mu_\nu\nabla^\alpha\phi\nabla_\alpha\phi\right) - \kappa^2V_{eff}\delta^\mu_\nu + X^\mu_\nu, \quad (19)$$

where the extra component X^μ_ν is defined as

$$X^\mu_\nu \equiv Y^\mu_\nu - \mathcal{Z}^\mu_\nu, \quad (20)$$

with

$$Y^\mu_\nu \equiv \frac{(2+n)\kappa^4}{4(3+n)}\sigma t^\mu_\nu + \kappa^4\pi^\mu_\nu, \quad (21)$$

and

$$\mathcal{Z}^\mu_\nu \equiv E^\mu_\nu + \frac{1}{(3+n)}\left(\nabla^\mu\phi\nabla_\nu\phi - \frac{1}{4+n}\delta^\mu_\nu\nabla^\alpha\phi\nabla_\alpha\phi\right). \quad (22)$$

Following Eq. (17) we have defined the effective potential $V_{eff} = \Lambda_b/\kappa^2$, where the induced cosmological constant on the brane Λ_b is given by Eq. (15). Then the scalar field equation of motion becomes

$$\nabla^\alpha\nabla_\alpha\phi - V'_{eff} = J_n. \quad (23)$$

From Eq. (19), the Bianchi identity implies

$$\nabla_\mu X^\mu_\nu = -\kappa^2 J_n \nabla_\nu\phi. \quad (24)$$

Note that the energy-momentum tensor of the extra component is not conserved due to the existence of the bulk scalar field. In the absence of matter on the brane, $Y^\mu_\nu = 0$, then we have $X^\mu_\mu = -\mathcal{Z}^\mu_\mu = 0$. In this case, we can interpret the extra component as the energy-momentum tensor for the dark radiation.

In the following section, we attempt to study analytically the cosmological consequences of higher-dimensional brane-world. We used the extra component formalism to discuss cosmology in the brane Universe in the context of the Kaluza-Klein brane-world scheme, i.e., to consider Kaluza-Klein compactifications on the brane.

III. DILATONIC KALUZA-KLEIN BRANE-WORLD COSMOLOGY

For the cosmological applications, we are interested in homogeneous and isotropic geometries on the brane, hence the metric on the brane is taken as the Friedmann-Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j + b^2(t)\delta_{\alpha\beta}dz^\alpha dz^\beta, \quad (25)$$

where δ_{ij} represents the metric of three-dimensional ordinary spaces with the spatial coordinates x^i ($i = 1, 2, 3$), while $\delta_{\alpha\beta}$ represents the metric of n -dimensional compact spaces with the coordinates z^α ($\alpha = 4, \dots, 3+n$). We assume that the internal space is given by n -dimensional torus. The scale factor b denotes the size of the internal dimensions, while the scale factor a is the usual scale factor for the external space.

In the background metric (25) and assuming that ϕ

only depends on time, we obtain the following equations

$$G^0_0 = -\kappa^2 \left(\frac{1}{2} \dot{\phi}^2 + V_{eff} \right) + X^0_0, \quad (26)$$

$$G^i_j = \kappa^2 \left(\frac{1}{2} \dot{\phi}^2 - V_{eff} \right) \delta^i_j + X^i_j, \quad (27)$$

$$G^\alpha_\beta = \kappa^2 \left(\frac{1}{2} \dot{\phi}^2 - V_{eff} \right) \delta^\alpha_\beta + X^\alpha_\beta, \quad (28)$$

$$\ddot{\phi} + 3H_a \dot{\phi} + nH_b \dot{\phi} + V'_{eff} = -J_n, \quad (29)$$

$$\dot{t}^0_0 + 3H_a (t^0_0 - t^1_1) + nH_b (t^0_0 - t^4_4) = 0, \quad (30)$$

where $H_a = \dot{a}/a$ and $H_b = \dot{b}/b$, and the components of the $(4+n)$ -dimensional Einstein tensor are

$$G^0_0 = -3 \left(H_a^2 + \frac{k_a}{a^2} \right) - n \left[\frac{(n-1)}{2} H_b^2 + 3H_a H_b + \frac{(n-1)}{2} \frac{k_b}{b^2} \right], \quad (31)$$

$$G^i_j = - \left(2\dot{H}_a + 3H_a^2 + \frac{k_a}{a^2} \right) \delta^i_j - n \left[\dot{H}_b + \frac{(n+1)}{2} H_b^2 + 2H_a H_b + \frac{(n-1)}{2} \frac{k_b}{b^2} \right] \delta^i_j, \quad (32)$$

$$G^\alpha_\beta = -3 \left(\dot{H}_a + 2H_a^2 + \frac{k_a}{a^2} \right) \delta^\alpha_\beta - (n-1) \left[\dot{H}_b + \frac{n}{2} H_b^2 + 3H_a H_b + \frac{(n-2)}{2} \frac{k_b}{b^2} \right] \delta^\alpha_\beta. \quad (33)$$

The values of k_a and k_b are related to the curvatures of the external space and the internal space, respectively. In this paper we assume $k_b = 0$ for simplicity. Equations (26)-(30) are our basic equations to study cosmology in the Kaluza-Klein brane-world.

The constraint equation for the extra component is given by

$$\dot{X}^0_0 + 3H_a (X^0_0 - X^1_1) + nH_b (X^0_0 - X^4_4) = -\kappa^2 J_n \dot{\phi}. \quad (34)$$

By combining Eqs. (26), (27), and (28) one can write

$$X^0_0 + 3X^1_1 - 2X^4_4 = G^0_0 + 3G^1_1 - 2G^4_4 + 2\kappa^2 V_{eff}. \quad (35)$$

A. Solutions with $\Lambda_b = 0$ and $J_n = 0$

Let us first consider that the bulk and brane potentials obey the generalized Randall-Sundrum condition $\Lambda_b = 0$, so that $V_{eff} = 0$, and the energy conservation for scalar field on the brane is satisfied, $J_n = 0$. The solution of Eq. (29) is given by

$$\dot{\phi} = \frac{c_\phi}{a^3 b^n}, \quad (36)$$

where c_ϕ is an integration constant. Note that the scalar field depends on both the external scale factor and the internal scale factor, $\phi(t) = \phi(a(t), b(t))$.

In the following we consider a special case in which the brane-world evolves with two scale factors. We take a simple relation between the scale factors on the brane of the form $b(t) = a^\gamma(t)$, where γ is a constant. For the internal scale factor $b(t)$ to be small compared to

the external scale factor $a(t)$, the constant γ should be negative. In this choice we have $H_b = \gamma H_a \equiv \gamma H$, and then Eq. (34) becomes

$$\dot{X}^0_0 + (4 + n\gamma) H X^0_0 = -n\gamma(1 - \gamma) \left[\dot{H} + (3 + n\gamma) H^2 \right] H. \quad (37)$$

The Friedmann equation is given by

$$3(1 + \alpha_0) H^2 = -3 \frac{k_a}{a^2} + \frac{\kappa^2}{2} \frac{c_\phi^2}{a^{2(3+n\gamma)}} - X^0_0, \quad (38)$$

where

$$\alpha_0 = \frac{n\gamma(6 - \gamma + n\gamma)}{6}. \quad (39)$$

The extra component X^0_0 is determined by the solution of the equation (37) which can be rewritten as

$$\dot{X}^0_0 + (4 + \beta_0) X^0_0 H = \beta_1 \frac{k_a}{a^2} H, \quad (40)$$

where

$$\begin{aligned} \beta_0 &= \frac{n\gamma(2 + \gamma + n\gamma)}{3 + n\gamma}, \quad \beta_1 = \frac{6n\gamma(1 - \gamma)}{3 + n\gamma}, \\ \beta_2 &= \frac{3n(1 - \gamma)(n - 1)(4 + \gamma + 3n\gamma)}{4(3 + n\gamma)}. \end{aligned} \quad (41)$$

Note that the effect of internal dimension is given by definitions (39) and (41). Equation (40) can be integrated to gives

$$X^0_0 = \frac{\beta_1}{2 + \beta_0} \frac{k_a}{a^2} - \frac{\varepsilon_0}{a^{4+\beta_0}}, \quad (42)$$

where ε_0 is an integration constant. Inserting Eq. (42) into Eq. (38), we find the modified Friedmann equation

$$3(1 + \alpha_0)H^2 = -3 \left[1 + \frac{\beta_1}{3(2 + \beta_0)} \right] \frac{k_a}{a^2} + \frac{\kappa^2}{2} \frac{c_\phi^2}{a^{2(3+n\gamma)}} + \frac{\varepsilon_0}{a^{4+\beta_0}}. \quad (43)$$

In what follows we study the solutions with an initial singularity ($a = 0$). Using the conformal time variable η defined by the differential relation $dt = ad\eta$, we can rewrite Eq. (43) as

$$3(1 + \alpha_0) \left(\frac{da}{d\eta} \right)^2 = -3 \left[1 + \frac{\beta_1}{3(2 + \beta_0)} \right] k_a a^2 + \frac{\kappa^2}{2} \frac{c_\phi^2}{a^{2(1+n\gamma)}} + \frac{\varepsilon_0}{a^{\beta_0}}, \quad (44)$$

and Eq. (36) as

$$\frac{d\phi}{d\eta} = \frac{c_\phi}{a^{(2+n\gamma)}}. \quad (45)$$

In the following we consider three cases: $\gamma = 0, \pm 1$.

In the case $\gamma = -1$, the internal scale factor $b(t)$ is proportional to $b(t) = 1/a(t)$. We have $a \rightarrow 0$, and the infinitely large internal space, $b \rightarrow \infty$.

In the case $\gamma = 1$, the internal scale factor $b(t)$ is related to $a(t)$ as $b(t) = a(t)$. The external scale factor evolves as

$$a^{2+n}(\tau) = \frac{\varepsilon_0 \tau (\tau + \tau_c)}{3(1 + k_a \tau^2)}, \quad (46)$$

where the new variable time is given by

$$\tau(\eta) = \begin{cases} \sqrt{\frac{3(2+n)}{2(3+n)}} |\eta|, & \text{for } k_a = 0 \\ \tan \sqrt{\frac{3(2+n)}{2(3+n)}} |\eta|, & \text{for } k_a = +1 \\ \tanh \sqrt{\frac{3(2+n)}{2(3+n)}} |\eta|, & \text{for } k_a = -1 \end{cases}, \quad (47)$$

and

$$\tau_c = \frac{\sqrt{6\kappa} |c_\phi|}{\varepsilon_0}. \quad (48)$$

The scalar field evolves as

$$\phi - \phi_0 = \pm \sqrt{\frac{3+n}{(2+n)\kappa^2}} \ln \left| \frac{\tau}{\varepsilon_0(\tau + \tau_c)} \right|, \quad (49)$$

for $\varepsilon_0 \neq 0$, or

$$\phi - \phi_0 = \pm \sqrt{\frac{3+n}{(2+n)\kappa^2}} \ln |\tau|, \quad (50)$$

for $\varepsilon_0 = 0$, where \pm corresponds to the sign of c_ϕ . Note that the initial singularity appears at $\tau = 0$. If we convert

to proper time the evolution of the scale factor with $c_\phi \neq 0$ is given by

$$a(t) = \left(\frac{2}{3} \right)^{\frac{1}{2(2+n)}} \left(\frac{3+n}{2+n} \right)^{\frac{5+n}{2(2+n)(3+n)}} (\kappa c_\phi)^{\frac{1}{3+n}} t^{\frac{1}{3+n}}. \quad (51)$$

Then, the Universe starts expanding away from the initial singularity.

More interesting case is in the static internal dimension, $\gamma = 0$, we obtain

$$a^2(\tau) = \frac{\varepsilon_0 \tau (\tau + \tau_c)}{3(1 + k_a \tau^2)}, \quad (52)$$

where

$$\tau(\eta) = \begin{cases} |\eta|, & \text{for } k_a = 0 \\ \tan |\eta|, & \text{for } k_a = +1 \\ \tanh |\eta|, & \text{for } k_a = -1 \end{cases}, \quad (53)$$

and

$$\phi - \phi_0 = \pm \sqrt{\frac{3}{2\kappa^2}} \ln \left| \frac{\tau}{\varepsilon_0(\tau + \tau_c)} \right|, \quad (54)$$

for $\varepsilon_0 \neq 0$, or

$$\phi - \phi_0 = \pm \sqrt{\frac{3}{2\kappa^2}} \ln |\tau|, \quad (55)$$

for $\varepsilon_0 = 0$. In a spatially flat Universe the evolution of the scale factor is the same as the standard cosmological solution with stiff matter,

$$a(t) = \left(\frac{3}{2} c_\phi^2 \kappa^2 \right)^{1/6} t^{1/3}. \quad (56)$$

We also find that the Universe starts expanding away from the initial singularity.

B. Solutions with exponential potential

We may now consider the case when the bulk scalar field potential and the brane tension are given as exponents, with some constant parameters V_0 , σ_0 and λ ,

$$V(\phi) = V_0 \exp \left(-\frac{2}{3+n} \lambda \kappa \phi \right), \quad (57)$$

$$\sigma(\phi) = \sigma_0 \exp \left(-\frac{1}{3+n} \lambda \kappa \phi \right). \quad (58)$$

These are string-inspired values for the potentials [52–54]. Then the effective potential is given by

$$V_{eff} = V_{eff,0} \exp \left(-\frac{2}{3+n} \lambda \kappa \phi \right) = \frac{\Lambda_b}{\kappa^2}, \quad (59)$$

where

$$V_{eff,0} = \frac{2+n}{4+n} V_0 + \frac{(2+n)\kappa^2 \sigma_0^2}{8(3+n)} \left[1 - \frac{\lambda^2}{3(4+n)} \right]. \quad (60)$$

We also assume the proportionality relation between the scalar field and the logarithm of the scale factors,

$$\phi = \frac{\lambda}{\kappa} \ln(ab) = \frac{\lambda}{\kappa} \ln(a^{1+\gamma}). \quad (61)$$

From the above model, if $\lambda = 0$ we have $\phi = 0$ and then $V = V_0$, $\sigma = \sigma_0$, and $V_{eff} = V_{eff,0}$. The induced cosmological constant on the brane, Λ_b , is exactly constant. If we set $\Lambda_b = 0$, the generalized Randall-Sundrum condition is given by

$$\sigma_0 = \sqrt{\frac{8(3+n)}{(4+n)\kappa}}(-V_0). \quad (62)$$

For the positive brane tension, V_0 must be negative. Thus V_0 can be interpreted as a bulk cosmological constant and the Randall-Sundrum brane is a slice in $(5+n)$ -dimensional anti-de Sitter spacetime. For vanishing the extra component $X_{\mu\nu}$ or the Weyl tensor $E_{\mu\nu}$, we have the Minkowski spacetime on the brane.

Applying the above model, Eq. (34) becomes

$$\dot{X}_0^0 + (4 + \beta_{0\lambda}) X_0^0 H = \beta_{1\lambda} \frac{k_a}{a^2} H + \beta_3 \kappa^2 V_{eff} H, \quad (63)$$

where

$$\begin{aligned} \beta_{0\lambda} &= \beta_0 + \frac{(1+\gamma)^2}{3+n\gamma} \lambda^2, \quad \beta_{1\lambda} = \beta_1 - \frac{6(1+\gamma)^2}{3+n\gamma} \lambda^2, \\ \beta_{2\lambda} &= \beta_2 - \frac{(n-1)(3+5n)(1+\gamma)^2}{4(3+n\gamma)} \lambda^2, \\ \beta_3 &= \frac{6n(1-\gamma)^2}{(2+n)(3+n\gamma)} \\ &\quad + \frac{2(1+\gamma)(3+3n+n^2+4n\gamma+9\gamma)}{(2+n)(3+n)(3+n\gamma)} \lambda^2. \end{aligned} \quad (64)$$

The solution of Eq. (63) is given by

$$X_0^0 = \frac{\beta_{1\lambda}}{2+\beta_{0\lambda}} \frac{k_a}{a^2} + \frac{(3+n)\beta_3}{(3+n)(4+\beta_{0\lambda}) - 2\lambda^2} \kappa^2 V_{eff} - \frac{\varepsilon_{0\lambda}}{a^{(4+\beta_{0\lambda})}}, \quad (65)$$

where $\varepsilon_{0\lambda}$ is an integration constant. The Friedmann equation is given by

$$\begin{aligned} 3(1+\alpha_{0\lambda})H^2 &= -3 \left[1 + \frac{\beta_{1\lambda}}{3(2+\beta_{0\lambda})} \right] \frac{k_a}{a^2} \\ &\quad + \left[1 - \frac{(3+n)\beta_3}{(3+n)(4+\beta_{0\lambda}) - 2\lambda^2} \right] \frac{\kappa^2 V_{eff,0}}{a^{2\lambda^2/(3+n)}} \\ &\quad + \frac{\varepsilon_{0\lambda}}{a^{(4+\beta_{0\lambda})}}, \end{aligned} \quad (66)$$

where

$$\alpha_{0\lambda} = \alpha_0 - \frac{1}{6}(1+\gamma)^2 \lambda^2. \quad (67)$$

We also find that a possible energy leak out of the brane into the bulk is in the form

$$\begin{aligned} J_n &= -\frac{3(1+\alpha_{0\lambda})}{3+n\gamma} H \dot{\phi} - \frac{(1+\gamma)\lambda}{(3+n\gamma)\kappa} \\ &\quad \times \left[\frac{n(1+n) + \gamma(12+3n-n^2)}{(2+n)(3+n)(1+\gamma)} \frac{\kappa^2 V_{eff,0}}{a^{2\lambda^2/(3+n)}} - 3 \frac{k_a}{a^2} \right] \end{aligned} \quad (68)$$

Note that equations (66) and (68) include five-dimensional case, corresponding to $n = 0$ [54]. By contrast, in higher dimensions, J_n depends on the effective potential or the induced cosmological constant on the brane. For example, in the static internal dimension, $\gamma = 0$, and assuming $k_a = 0$ for simplicity, we find

$$J_n = - \left(1 - \frac{\lambda^2}{6} \right) H_a \dot{\phi} - \frac{n(1+n)\lambda}{3(2+n)(3+n)\kappa} \Lambda_b, \quad (69)$$

where we have used $\kappa^2 V_{eff} = \Lambda_b$. If we impose the generalized Randall-Sundrum condition, $\Lambda_b = 0$, the energy is flowing onto the brane when $\lambda^2 < 6$, and flowing out to the brane when $\lambda^2 > 6$. For $\Lambda_b > 0$, the energy is flowing onto the brane when $0 < \lambda < \sqrt{6}$, and flowing out to the brane when $\lambda < -\sqrt{6}$. While for $\Lambda_b < 0$, the energy is flowing onto the brane when $-\sqrt{6} < \lambda < 0$, and flowing out to the brane when $\lambda > \sqrt{6}$.

IV. STATIC INTERNAL DIMENSION SOLUTIONS WITH MATTER ON THE BRANE

In the previous section we have focussed upon dilaton-vacuum solutions on the brane. For completeness we now study the presence of matter on the brane. In this case we do not have $X^\mu_\mu = 0$. Indeed, we have $\mathcal{Z}^\mu_\mu = 0$ which it implies

$$\mathcal{Z}_0^0 + 3\mathcal{Z}_1^1 + n\mathcal{Z}_4^4 = 0. \quad (70)$$

Using Eq. (20) and by combining Eqs. (35) and (70) we find

$$\begin{aligned} -n\mathcal{Z}_4^4 &= \mathcal{Z}_0^0 + 3\mathcal{Z}_1^1 = -\frac{n}{2+n} (G_0^0 + 3G_1^1 - 2G_4^4) \\ &\quad - \frac{2n}{2+n} \kappa^2 V_{eff} + \frac{n}{2+n} (Y_0^0 + 3Y_1^1 - 2Y_4^4), \end{aligned} \quad (71)$$

where Y^μ_ν is given by Eq. (21) with the components of π^μ_ν are

$$\pi_0^0 = -\frac{1}{8(3+n)} \left\{ 2(t_0^0)^2 + n \left[(t_0^0)^2 - 3(t_1^1 - t_4^4)^2 \right] \right\}, \quad (72)$$

$$\pi_j^i = \frac{1}{8(3+n)} \left\{ 2(t_0^0)^2 - 4t_0^0 t_1^1 + n \left[(t_0^0)^2 - 2t_0^0 t_4^4 + (t_1^1 - t_4^4)(t_1^1 - 3t_4^4) \right] \right\} \delta_j^i, \quad (73)$$

$$\pi_\beta^\alpha = \frac{1}{8(3+n)} \left\{ 2(t_0^0)^2 - 6t_0^0 t_1^1 + 2t_4^4 (t_0^0 + 3t_1^1 - 3t_4^4) + n \left[(t_0^0)^2 - 2t_0^0 t_4^4 + 3(t_1^1 - t_4^4)^2 \right] \right\} \delta_\beta^\alpha. \quad (74)$$

Here $-t_0^0$ is the total energy density, $t_1^1 = t_2^2 = t_3^3$ the total external pressure and $t_4^4 = t_5^5 = \dots = t_{3+n}^{3+n}$ the total internal pressure. Inserting Eq. (20) into Eq. (34) and then eliminating \mathcal{Z}_1^1 and \mathcal{Z}_4^4 by using Eq. (71), we find

$$\begin{aligned} \dot{\mathcal{Z}}_0^0 + (4H_a + nH_b) \mathcal{Z}_0^0 &= \kappa^2 J_n \dot{\phi} \\ -\frac{n}{2+n} (G_0^0 + 3G_1^1 - 2G_4^4 + 2\kappa^2 V_{eff}) (H_a - H_b) \\ + \dot{Y}_0^0 + \frac{2}{2+n} [(3+2n)Y_0^0 - 3Y_1^1 - nY_4^4] H_a \\ + \frac{n}{2+n} [(1+n)Y_0^0 - 3Y_1^1 - nY_4^4] H_b. \end{aligned} \quad (75)$$

Note that in the absence of matter on the brane, $\mathcal{Z}_\nu^\mu = -X_\nu^\mu$, Eq. (75) reduced to Eq. (34).

In the following we study the static internal dimension case for simplicity. In this case we have

$$H_b = 0, \quad \text{and} \quad G_0^0 + 3G_1^1 - 2G_4^4 = 0. \quad (76)$$

Then, Eq. (75) becomes

$$\begin{aligned} \dot{\mathcal{Z}}_0^0 + 4\mathcal{Z}_0^0 H_a &= \kappa^2 J_n \dot{\phi} - \frac{2n}{2+n} \kappa^2 V_{eff} H_a + \dot{Y}_0^0 \\ + \frac{2}{2+n} [(3+2n)Y_0^0 - 3Y_1^1 - nY_4^4] H_a. \end{aligned} \quad (77)$$

The conservation of the energy-momentum tensor for the matter field on the brane and the equation of motion of the scalar field, respectively, are given by

$$\dot{t}_0^0 = -3H_a (t_0^0 - t_1^1), \quad (78)$$

$$\ddot{\phi} + 3H_a \dot{\phi} + V'_{eff} = -J_n. \quad (79)$$

Eliminating Y_ν^μ in Eq. (77), we obtain

$$\begin{aligned} \dot{\mathcal{Z}}_0^0 + 4\mathcal{Z}_0^0 H_a &= \kappa^2 J_n \dot{\phi} - \frac{2n}{2+n} \kappa^2 V_{eff} H_a \\ + \frac{(2+n)\kappa^4}{4(3+n)} \sigma' \dot{\phi} t_0^0 + \frac{n\kappa^4}{4(3+n)} \sigma [t_0^0 + 3t_1^1 - 2t_4^4] H_a \\ + \kappa^4 \dot{\pi}_0^0 + \frac{2}{2+n} [(3+2n)\pi_0^0 - 3\pi_1^1 - n\pi_4^4] H_a. \end{aligned} \quad (80)$$

We now consider the exponential potential as discussed in the previous section with static internal dimension. By defining the equations of state $t_1^1 = -w_i t_0^0$, $t_4^4 = -w_\alpha t_0^0$, and inserting Eq. (79) into Eq. (80) for J_n , one can integrate the following equation,

$$\begin{aligned} \dot{\mathcal{Z}}_0^0 + \left(4 + \frac{\lambda^2}{3} \right) \mathcal{Z}_0^0 H_a &= -\frac{6n(3+n) + 2(3+3n+n^2)b^2}{3(2+n)(3+n)} \kappa^2 V_{eff} H_a + 2\lambda^2 \frac{k_a}{a^2} H_a \\ + \frac{3n(3+n)(1-3w_i+2w_\alpha) + [3+3n+n^2 - (3+n)(3w_i+nw_\alpha)]\lambda^2}{12(3+n)^2} \kappa^4 \sigma t_0^0 H_a \\ + \frac{3n(1-3w_i+3w_\alpha)(w_\alpha+3(1+w_i)(w_i-w_\alpha)) + [3w_i+nw_\alpha+3n(w_i-w_\alpha)^2]\lambda^2}{12(3+n)} \kappa^4 (t_0^0)^2 H_a, \end{aligned} \quad (81)$$

and yields

$$\begin{aligned}
\mathcal{Z}_0^0 = & -\frac{6n(3+n) + 2(3+3n+n^2)\lambda^2}{(2+n)[12(3+n) + (n-3)\lambda^2]} \kappa^2 V_{eff} \\
& - \frac{3n(3+n)(1-3w_i+2w_\alpha) + [3+3n+n^2 - (3+n)(3w_i+nw_\alpha)]\lambda^2}{4(3+n)[3(3+n)(1-3w_i) + n\lambda^2]} \kappa^4 \sigma t_0^0 \\
& + \frac{3n(1-3w_i+3w_\alpha)[w_\alpha + 3(1+w_i)(w_i-w_\alpha)] + [3w_i+nw_\alpha + 3n(w_i-w_\alpha)^2]\lambda^2}{4(3+n)[6(1+3w_i) - \lambda^2]} \kappa^4 (t_0^0)^2.
\end{aligned} \tag{82}$$

Inserting Eq. (82) into (00)-component of the Einstein equations we find

$$H_a^2 + \frac{\bar{k}_a}{a^2} = \beta_4 \kappa^2 V_{eff} + 8\pi G_{eff} \rho + \beta_5 \rho^2 + \frac{\bar{C}}{a^{4+\lambda^2/3}}, \tag{83}$$

$$\begin{aligned}
J_n = & - \left(1 - \frac{\lambda^2}{6}\right) H_a \dot{\phi} + \frac{\lambda k_a}{\kappa a^2} - \frac{n(1+n)\lambda\kappa}{3(2+n)(3+n)} V_{eff} - \frac{(1-3w_i-nw_\alpha)\lambda\kappa^3}{12(3+n)} \sigma \rho \\
& + \frac{[4(1+3w_i) + 2n(2+2w_i+3w_\alpha+3(w_i-w_\alpha)^2) + n^2(1+2w_i+3(w_i-w_\alpha)^2)]\lambda\kappa^3}{24(2+n)(3+n)} \rho^2,
\end{aligned} \tag{84}$$

where $\rho = -t_0^0$, and

$$\bar{k}_a = \left(1 + \frac{\lambda^2}{6}\right)^{-1} k_a, \tag{85}$$

$$\beta_4 = \frac{2(3+n)(4+n)}{(2+n)[12(3+n) + (n-3)\lambda^2]}, \tag{86}$$

$$8\pi G_{eff} = \frac{(1-3w_i-nw_\alpha)}{2(3+n)(1-3w_i) + 2n\lambda^2} \kappa^4 \sigma, \tag{87}$$

$$\beta_5 = \frac{2(1+3w_i) + n(1+2w_\alpha) + 3(w_i-w_\alpha)^2}{4(3+n)[6(1+3w_i) - \lambda^2]} \kappa^4 \tag{88}$$

$$\bar{C} = \frac{1}{3} \left(1 - \frac{\lambda^2}{6}\right)^{-1} C. \tag{89}$$

The first two terms on the right hand side of (83) are what we would expect for standard four dimensional cosmology. The third term is quadratic in the brane energy momentum tensor, and the fourth term is a generalized dark radiation energy component. Note that in contrast with five-dimensional case, the effective gravitational constant depends on time through the brane tension and the equation of state. One can obtain the variation rate of the effective gravitational constant is

$$\frac{\dot{G}_{eff}}{G_{eff}} = -\frac{\lambda^2}{3+n} H_a. \tag{90}$$

We see that the variation rate of the effective gravitational constant is smaller than five-dimensional case ($n = 0$) with the exponential potential [54], and it is relatively small if the number of the internal dimension

n is large. Observational bounds on \dot{G}_{eff}/G_{eff} then constrain the parameters of the theory. If we assume $|\dot{G}_{eff}/(G_{eff}H_{a,0})| \lesssim 10^{-2}$, where $H_{a,0}$ is the value of the Hubble parameter today, we have $\lambda^2 \lesssim (3+n)/100$.

V. COSMOLOGICAL EVOLUTION OF THE SOURCES IN THE EINSTEIN FRAME

In the previous sections, we have discussed the cosmological evolutions in the induced metric frame, which is given by Eq. (14). In this section, we study the 4-dimensional cosmological evolution in the Einstein frame and discuss the properties of the resulting 4-dimensional cosmological evolution in this frame. A complementary way of studying higher-dimensional theories is to dimensionally reduce the action by integrating out the internal dimensions and then perform a conformal transformation.

From Eq. (14) we define the Newton gravitational constant in $(4+n)$ -dimensions as

$$8\pi G_{4+n} = \frac{(2+n)\kappa^4}{4(3+n)} \sigma. \tag{91}$$

If the brane tension is not depending on the bulk scalar field (minimally coupled), the $(4+n)$ -dimensional Newton gravitational constant is truly constant. In this section we assume $\sigma = const$. Note that recovering the 4-dimensional effective Newton gravitational constant re-

quires an additional time-dependent factor,

$$8\pi G_4 = \frac{8\pi G_{4+n}}{V_n}, \quad (92)$$

where V_n denotes the volume of the n -dimensional internal spaces, while the 4-dimensional cosmological constant is given by

$$\Lambda_4 = V_n \Lambda_b, \quad (93)$$

where Λ_b is given by Eq. (15). Note that both G_4 and Λ_4 are time-dependent. However, it must be remembered that we are not in the Einstein frame. Similarly, the 4-dimensional energy density is given by $\rho_4 = V_n \rho$.

One can compare the time-dependent of the effective Newton gravitational constant Eqs. (87) and (92). We find

$$G_4 = \frac{(2+n)[2(3+n)(1-3w_i) + 2n\lambda^2]}{4(3+n)(1-3w_i - nw_\alpha)} \frac{G_{eff}}{V_n}. \quad (94)$$

Here, V_n to be a constant, which corresponding to the static internal dimensions. As we expected

$$\frac{\dot{G}_4}{G_4} = \frac{\dot{G}_{eff}}{G_{eff}}. \quad (95)$$

However, for the static internal dimensions case with a constant brane tension, the effective Newton gravitational constant is exactly constant. It depends on the equations of state, the potential parameter λ and the number of internal dimensions.

In order to recovering 4-dimensional cosmological evolution in the Einstein frame, we proceed through three steps: (1) a dimensional reduction of the $(4+n)$ -action by integrating out the internal dimensions; (2) a conformal transformation of the 4-dimensional induced metric into Einstein frame; (3) a redefinition of the scale factor b into a new scalar field to give its kinetic term a canonical form. From the gravitational sector, the resulting 4-dimensional Einstein frame action is given by

$$S_* = \int d^4x \sqrt{-h_*} \left[\frac{1}{16\pi G_{4*}} R_* - \frac{1}{2} h^{*\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V_*(\Phi) \right], \quad (96)$$

where we mark the quantities in the Einstein frame with the subscript $*$ and a new scalar field is defined by

$$\Phi = \sqrt{\frac{n(2+n)}{16\pi G_{4*}}} \ln b. \quad (97)$$

We also have the relations between the variables in the 4-dimensional induced metric frame and those in the Einstein frame as

$$dt_* = b^{n/2} dt, \quad a_* = b^{n/2} a, \quad (98)$$

where the scale factor b is appropriately expressed in Φ . Hence we have the relation

$$H_{a*} = b^{-n/2} \left(H_a + \frac{n}{2} H_b \right). \quad (99)$$

where H_{a*} is the Hubble parameter in the Einstein frame,

$$H_{a*} = \frac{1}{a_*} \frac{da_*}{dt_*}. \quad (100)$$

In the static internal dimensions case, $b = b_0 = 1$, we have $a_* = a$ so that $H_{a*} = H_a$.

From the matter sector, we have

$$\rho_* = b^{-2n} \rho_4. \quad (101)$$

Assuming $V_n \sim b^n$, the evolution of the 4-dimensional energy density in the Einstein frame is given by

$$\rho_* \propto b^{-\frac{n}{2}(1-3w_i+2w_\alpha)} a_*^{-3(1+w_i)}. \quad (102)$$

Note that we recover standard 4-dimensional energy density for $n = 0$ or if the equations of state satisfy the following constraint:

$$1 - 3w_i + 2w_\alpha = 0. \quad (103)$$

For instance, in the radiation-dominated era $w_i = 1/3$, we find $w_\alpha = 0$, the internal pressure drops to zero. Moreover, in order to understanding the physical interpretations of the constraint (103), let us consider equation (76). Since the components of the Einstein tensor are corresponding to the matter field: $G^0_0 \sim t^0_0$, $G^1_1 \sim t^1_1$, and $G^4_4 \sim t^4_4$, and using the relations $t^1_1 = -w_i t^0_0$, $t^4_4 = -w_\alpha t^0_0$, the constraint (103) can be interpreted as a necessary condition for static internal dimensions.

We have shown that we can obtain the standard cosmology in the Einstein frame by assuming a constant brane tension. In the case of the non-minimally coupled to bulk scalar field, in which the brane tension is a function of the bulk scalar field, a conformal transformation is determined by the form of brane tension. For instance, if the brane tension is given by Eq. (58), the Einstein frame is obtained by a conformal transformation of the 4-dimensional metric: $h_{*\mu\nu} = b^n e^{2\varphi} h_{\mu\nu}$, where $2\varphi = \lambda \kappa \phi / (3+n)$. Then the Hubble parameter in the Einstein frame is related to the Hubble parameter in the induced metric frame as

$$H_{a*} = b^{-n/2} e^{-\varphi} \left(H_a + \frac{n}{2} H_b + \dot{\varphi} \right), \quad (104)$$

and the evolution of the 4-dimensional energy density in the Einstein frame is given by $\rho_* = b^{-n} e^{-4\varphi} \rho$, with ρ is the higher-dimensional energy density.

VI. CONCLUSIONS

In this paper we studied the brane-world cosmological model in higher-dimensional spacetime with a bulk scalar field. The $(4+n)$ -dimensional gravitational field equations can be formulated to standard form with the extra component. In the absence of the matter on the brane, we can interpret the extra component as the energy-momentum tensor for the dark radiation. We used this

formalism to discuss cosmology in the brane Universe in the context of the Kaluza-Klein brane-world scheme, i.e., to consider Kaluza-Klein compactifications on the brane. By assuming the cosmological symmetry on the Kaluza-Klein brane, we derived the Friedmann equation and a possible energy leak out of the brane into the bulk.

If the bulk and brane potentials obey the generalized Randall-Sundrum condition and the energy conservation for scalar field on the brane is satisfied, we find that the brane Universe starts expanding away from the initial singularity both for the internal scale factor is proportional to the external scale factor and the static internal scale factor. A significant result in present study is that the possibility of energy flow out of the brane into the bulk, which it depends on the effective potential or the effective cosmological constant on the brane. It is different with the five-dimensional dilatonic brane-world cosmologies. By constraining the potential parameter λ and the effective cosmological constant, there exist two possibilities of the energy flows: onto and out to the brane. Finally, we derive the Friedmann equation with matter on the brane. We find that the effective gravitational constant depends on time through the brane tension and

the equation of state. However, after appropriate conformal transformation, the 4-dimensional Newton gravitational constant in the Einstein frame is true constant. A new result in Section V is that we recover standard 4-dimensional energy density in the Einstein frame if the equations of state satisfy the constraint Eq. (103).

There would be various extensions of our considerations. By introducing a bulk scalar field, it is interesting to stabilize the distance between the two Kaluza-Klein branes as is done in the context of the first model introduced by Randall and Sundrum. It also important to understand inflation without inflaton and the quantum fluctuations of brane-inflation in the Kaluza-Klein brane scenario with a bulk scalar field. It is intriguing to consider the low energy description of this brane-world model [51]. We leave these issues for future studies.

VII. ACKNOWLEDGEMENT

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